Variability of rain drop size distribution and its effect on the Z–R relationship: A case study for intense Mediterranean rainfall

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Abstract

This paper presents an analysis of the variability of rain drop size distributions in intense Mediterranean rainfall and its impact on the reflectivity—rain-rate conversion. Two concurrent approaches for estimating the Z–R relationship from DSD measurements are reviewed: (1) non-linear regression techniques on the scattergraphs of the (Z, R) pairs derived for each DSD spectra; (2) use of a DSD model fitting based on a scaling law formulation. The two approaches are implemented over a DSD dataset of 75 h of Mediterranean rain collected with a ground-based optical DSD sensor. As a result of the heterogeneity of the rain processes, the seasonal Z–R relationship coefficients obtained are very diverse and strongly dependent on the fitting methodology. A consistency test of the seasonal Z–R relationships is proposed to assess the most reliable estimation procedures in terms of rainfall estimation. Using the DSD-derived rain-rate time series as a reference, it is shown that the regression techniques are significantly better than the DSD modelling approach. The same consistency test shows that event-fitted Z–R relationships do not systematically improve the test scores, supporting the idea that the intra-event DSD variability is dominant. This finding is confirmed with an in-depth analysis of one rain event, showing evidence of rainfall organisation into several phases each one presenting very stable scale and shape DSD parameters over several hours, and abrupt transitions from one phase to the next. A rain-typing algorithm applied to the 3D reflectivity data observed concomitantly with an operational S-band radar is consistently able to recognise the two most intense phases of the rain event as convective.

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1. Introduction

Understanding the nature of rain drop size distribution variability, and determining whether it is random or associated with specific physical processes is fundamental in cloud physics. A good knowledge of the DSD and its space–time variability between and within precipitating systems is also important in hydrology, e.g. for radar quantitative precipitation estimation (QPE) and erosive process studies. Since the early work of Marshall and Palmer (1948), the DSD and the relation between the radar reflectivity (Z) and the rainfall intensity (R) is one centre of interest in the atmospheric community (Fujiwara, 1965; Waldvogel, 1974; Sempere Torres et al., 1994; Ulbrich and Atlas, 1997; Testud et al., 2001; Lee and Zawadzki, 2005; to name just a few contributions). Adapting the Z–R relationship to different rain types within a given storm is seen as a promising way to improve radar QPE. Waldvogel (1974), Tokay and Short (1996), Yuter and Houze (1997), Uijlhoet et al. (2003)
and Lee and Zawadzki (2005) are among authors who have studied the DSD measured at ground level and tried to link its characteristics to the different rain types – convective or stratiform – within a storm. Other authors (Steiner et al., 1995; Sanchez-Diezma et al., 2000; Delrieu et al., in preparation) have proposed ways to distinguish convective and stratiform rain on the basis of their 3D structure, analysed using volumetric radar data.

This paper is a preliminary contribution to this topic within the Cévennes-Vivarais Mediterranean Hydro-meteorological Observatory (OHM-CV is the French acronym, http://www.lthe.hmg.inpg.fr/OHM-CV/index.php), a research observatory devoted to the observation, modelling and forecasting of heavy rainfall and subsequent flash-flood events (Delrieu et al., 2005). More specifically, the paper presents the analysis of the variability of rain drop size distributions (DSD) in intense Mediterranean rainfall and its impact on the reflectivity — rain-rate conversion. The DSD dataset was collected during the autumn 2004 with a ground-based DSD sensor to start documenting Mediterranean DSD and defining a permanent DSD observation strategy for the OHM-CV. The dataset available is described in Section 2. Section 3 is devoted to the theoretical background concerning DSD modelling and the various methodologies that can be used for the derivation of the \( Z-R \) relationship. A sensitivity test to the methodology and to various parameters (integration time step, rain-rate threshold) is proposed in Section 4 by using DSD-derived rain-rate time series as reference. In Section 5, the intra-event variability of the DSD is studied for the most intense rain event of the season and a preliminary attempt is proposed to link the DSD data characteristics at ground level to the 3D radar reflectivity observed concomitantly with an S-band radar.

2. Dataset

The Cévennes-Vivarais region, located in the Mediterranean part of France (Fig. 1), is prone to intense rain events and subsequent flash-floods during the autumn season (e.g., Delrieu et al., 2005). The pilot site of the Cévennes-Vivarais Mediterranean Hydro-meteorological Observatory has a size of 32,000 km\(^2\) and is equipped with a full set of hydrological and meteorological operational and research instruments, including a dense network of rain and stream level gauges. It is also covered by two S-band weather radar systems, located near the cities of Bollène and Nîmes which belong to the Météo-France ARAMIS operational network. Since 2002, a volume-scanning protocol has been implemented for these radars in a semi-operational mode to provide 3D observations of the atmosphere (Delrieu et al., in preparation; Boudevillain et al., in preparation).

During the autumn 2004, an OTT/Parsivel DSD sensor was set up at Alès, i.e. at 40 km and 57 km of the Nîmes and Bollène radar sites, respectively. The DSD sensor (Löffler-Mang and Joss, 2000) is based on an optical principle with a near-IR (780 nm) rectangular single-beam of 27 mm wide, 180 mm long (that is a sensor horizontal area of 48.6 cm\(^2\)) and 10 mm high. It archives equivalent drop diameters sorted into 32 diameter classes from 0 up to 26 mm with a variable diameter increment (0.125 up to 3 mm). The time resolution can be selected and was set to 1 min in the present experiment. Although terminal velocity measurements are also archived by this sensor, such data were found to be too noisy here for a reliable utilization. We therefore consider in the following a model for the terminal velocity of raindrop as function of their diameter.

Five events with rain total amounts greater than 20 mm were observed in Alès during the autumn 2004 and selected for the present study. Table 1 gives additional information regarding the duration (9 to 22 h), the maximum one-minute rain rate (50 to 80 mm h\(^{-1}\)) and the total rain amount (up to 100 mm) of each event. Such figures are far from exceptional in the region. They are typical of the shallow convection that occurs within the warm sector of Mediterranean perturbations when hot and moist air originating from the Mediterranean Sea is lifted by the Cévennes relief.

Fig. 1. Location of the pilot site of the Cévennes-Vivarais Mediterranean Hydro-meteorological Observatory in France (top-left graph) together with the topography and locations of the DSD sensor (Alès) and the two Météo-France S-band weather radars in Nîmes and Bollène with 50-km range markers (right graph). The DSD sensor was located at the transition of the mountainous and plain parts of the Cévennes-Vivarais region.

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Table 1
Some characteristics of the five rain events selected

<table>
<thead>
<tr>
<th>Date</th>
<th>N spectra</th>
<th>Max R (mm h⁻¹)</th>
<th>RA (mm)</th>
<th>RE (%)</th>
<th>Z Bias (dBZ)</th>
<th>Z Cor</th>
</tr>
</thead>
<tbody>
<tr>
<td>14/10/2004</td>
<td>533</td>
<td>82</td>
<td>42.5</td>
<td>28.0</td>
<td>No data</td>
<td>No data</td>
</tr>
<tr>
<td>18/10/2004</td>
<td>1117</td>
<td>80</td>
<td>71.5</td>
<td>−2.7</td>
<td>2.2</td>
<td>0.27</td>
</tr>
<tr>
<td>25/10/2004</td>
<td>1008</td>
<td>53</td>
<td>35.8</td>
<td>−5.7</td>
<td>4.8</td>
<td>0.36</td>
</tr>
<tr>
<td>27/10/2004</td>
<td>1309</td>
<td>78</td>
<td>100.7</td>
<td>−9.1</td>
<td>−0.2</td>
<td>0.54 (0.71)</td>
</tr>
<tr>
<td>03/11/2004</td>
<td>578</td>
<td>60</td>
<td>28.3</td>
<td>11.0</td>
<td>3.9</td>
<td>0.51</td>
</tr>
</tbody>
</table>

N spectra is the number of non-zero one-minute DSD spectra, and gives as such an estimate of the event duration in minutes. Max R is the one-minute maximum rain rate (mm h⁻¹). RA (mm) is the DSD-measured event total rain amount. RE (%) is the relative error on the event total rain amount evaluated with respect to an operational rain gauge located at 3 km from the DSD sensor; Z Bias and Z Cor are the difference of the means (expressed in dBZ) and the correlation coefficient, respectively, of the DSD-derived reflectivity and radar-measured reflectivity time series. The radar reflectivity time series, based on one PPI every 5 min, come from the Bollène weather radar located at 57 km from the DSD-sensor site. The correlation coefficient value in brackets for the 27–28 October 2004 case was obtained for the reflectivity time series derived from an 8-elevation angle scanning strategy.

During the autumn 2004, the weather radars were mostly operated with their basic protocols, made of 3 PPIs performed every 5 min. The available processed data correspond to a pseudo-CAPPI with a range-dependent choice of a single elevation among the three measurements available in the vertical. For instance, at the ranges of the DSD sensor, the operational radar products are based on the 1.3° and 1.2° elevation angles for the Nîmes and Bollène radars, respectively. Volume-scanning measurements performed with the Bollène radar are available for one among the five events, the 27–28 October 2004 case, which is fortunately the most intense event of the season. For this case, the scanning strategy is made of 8 elevation angles with a revisiting time of 5 min. Details regarding the radar characteristics can be found in Delrieu et al. (in preparation) and will not be reproduced here for the sake of conciseness.

Table 1 gives the results of consistency checks that were realized to assess the quality of the DSD-sensor estimates. The first check was done by comparing the DSD-derived and rain gauge-derived total rain amounts. Note that the closest rain gauge was located at 3 km from the DSD-sensor site, leaving a rather important uncertainty in the assessment procedure related to the space variability of rainfall. However, one can notice that except for the first event, the relative error on the total rain amounts is less than 10% with no systematic over or under-estimation trend.

Visualisation of the radar rainfall maps (not shown here) confirms that such relative errors may be associated to the spatial variability of rainfall, especially for the first event for which strong rainfall gradients are present in the region of Alès. To overcome this ambiguity in the future, it has been decided to co-locate each DSD sensor with a rain gauge. A second check consists in comparing the DSD-derived and radar-measured reflectivity time series with a 5-min time resolution. Table 1 shows that the results in terms of bias and correlation are poor for the comparison with the Bollène radar; they are even worse for the Nîmes radar (not shown). Several factors may explain such discrepancies: e.g., radar calibration, screening effects (important for the Nîmes radar), reflectivity quantization effects... However, the main reason certainly lies in the sampling differences between the two sensors in terms of both spatial and temporal characteristics. In particular, we were able to show that the operational radar sampling strategy, with only one reflectivity sample every 5 min, has a very detrimental impact. The increased time resolution (8 samples every 5 min) of the volumetric radar data available for the 27–28 October 2004 event allowed to significantly improve the correlation coefficient between the radar and DSD-derived reflectivity time series for this case (0.71 instead of 0.54).

Due to these experimental limitations, the following work is mostly focused on the analysis of the DSD variability at ground level and its impact on the rain estimation via the use of Z–R relationships.

3. Theoretical background

Two main approaches can be used for estimating a Z–R relationship from DSD data:

3.1. Methodology 1

The first methodology is based on the derivation of a (Z, R) pair for each individual DSD spectra given a scattering model (Mie, Rayleigh...) and a raindrop terminal velocity model. A DSD spectra N(D) (dimension: [L⁻⁴]) is defined as the raindrop concentration in a given air volume as function of the diameter D[L]. The DSD spectra are evaluated from DSD measurements as:

\[ N(D)dD = n(D)/(v_t(D)SA)T \]  

where \( n(D) \) is the number of raindrops of diameter comprised between \([D - dD/2, D + dD/2] \), \( v_t(D) \) is the
raindrop terminal velocity \( [LT^{-1}] \), \( S \) the horizontal surface of the DSD sensor \( [L^2] \) and \( \Delta T \) the integration time step \( [T] \).

Using the Rayleigh scattering model, the subsequent radar reflectivity \( Z [L^6T^{-3}] \) can be expressed as:

\[
Z = C_Z \int_0^\infty N(D)D^6dD = C_Z \frac{S\Delta T}{C_0/C_1} \sum_{i=1}^{N_C} \left( n(D_i)D_i^6/v_i(D_i) \right) \tag{2}
\]

and the rain rate \( R [LT^{-1}] \) as:

\[
R = C_R \frac{\pi}{6} \int_0^\infty N(D)D^3v_i(D)dD = \frac{\pi C_R}{6S\Delta T} \sum_{i=1}^{N_C} \left( n(D_i)D_i^3 \right) \tag{3}
\]

where \( C_Z \) and \( C_R \) are numerical constants depending on the units chosen and \( N_C \) is the number of diameter classes. It is convenient to express the terminal velocity model as a power-law function of the diameter \( v_i(D)=cD^\beta \). We have chosen herein the model proposed by Atlas and Ulbrich (1977), \( v_i(D)=3.78D^{0.67} \), where \( D \) and \( v_i(D) \) are expressed in mm and m s\(^{-1} \), respectively. Once the \((Z, R)\) pairs are established from DSD time series, one can fit \( Z-R \) power-law models by using various non-linear regression techniques.

### 3.2. Methodology 2

The second methodology uses an intermediate step with the inference of a model for the DSD. DSD models are generally parameterized by the raindrop diameter and the rain rate. Therefore, one can derive the reflectivity as function of the rain rate given the models for the DSD, the electromagnetic scattering properties and the terminal velocities of the hydrometeors.

We consider hereafter the DSD modelling approach proposed by Sempere Torres et al. (1994) and further developed by Sempere Torres et al. (1998) and Uijlenhoet et al. (2003). This approach utilizes a scaling law formulation for the DSD with:

\[
N(D,R) = R^\alpha g(D/R^\beta) \tag{4}
\]

where \((\alpha, \beta)\) are the scale parameters and \( g(x) \) is a function describing the shape of the DSD. Note that other moments of the DSD, rather than the rain rate, could be considered in this formulation. The choice of \( R \) as the scaling variable is justified here by our modelling objective (establish a \( Z-R \) relationship).

Using (4), it can be shown that each moment of order \( n \) of the DSD can be written as a power law of the scaling variable \( R \), with:

\[
M_n = R^{\alpha+n\beta} \int_0^\infty x^n g(x)dx \tag{5}
\]

If one considers the rain rate as the \( M_n \) moment in (5), two constraints (self-consistency relations) on the DSD parameters can be derived which allow to reduce their degree of freedom:

\[
C_R \frac{\pi c}{6} \int_0^\infty x^{3+d} g(x)dx = 1 \tag{6a}
\]

\[
(4+d)\beta + \alpha = 1. \tag{6b}
\]

Practically, the inference of the DSD parameters is realized in two steps (illustrations are provided in Fig. 2):

1) the shape parameters \((\alpha, \beta)\) are estimated first: (i) \( \beta \) is evaluated as the slope of the regression line between the exponents of the \( n \)th order moments of the DSD as function of the scaling variable \( R \), according to (5), and (ii) \( \alpha \) is then evaluated with the self-consistency relation (6b).

2) the shape function \( g(x) \) is evaluated by plotting in a single graph the scaled spectra \( N(D)/R^\alpha \) as function of \( D/R^\beta \). Although one could work with non-parametric models for \( g(x) \), we choose here to fit two widely used functions with the negative exponential and the gamma models in order to be able to place our results with respect to the literature:

\[
g(D/R^\beta) = \kappa_1 \exp(-\lambda_1 D/R^\beta) \tag{7}
\]

\[
g(D/R^\beta) = \kappa_2 D^\mu \exp(-\lambda_2 D/R^\beta) \tag{8}
\]

where \((\kappa_1, \lambda_1)\) and \((\kappa_2, \mu, \lambda_2)\) are parameters fitted here with a moment technique combined with the self-consistency relation (6a).

Once the DSD parameters are estimated, the coefficients of the \( Z-R \) power-law model are calculated by introducing the reflectivity in (5), leading to:

\[
a_{ZR} = C_Z \int_0^\infty x^\beta g(x)dx \tag{9a}
\]

\[
b_{ZR} = \alpha + 7\beta. \tag{9b}
\]
4. Variability of the DSD at the season scale and impact on the Z–R relationship

We present hereafter an analysis of the DSD variability at the season scale and a sensitivity study of the Z–R relationship on various factors and parameters. For this purpose, we have grouped the DSD spectra of the five rain events selected during the autumn 2004, corresponding to about 75 rainy hours.

4.1. Variants for the Z–R relationship estimation

The two methodologies described in Section 3 are implemented. For Methodology 1 (M1 hereafter), the two variants considered differ by the regression technique used: (i) linear regression on log-transformed values, termed as M1/LOGREG, and (ii) non-linear regression using a Newton–Raphson search procedure for the exponent of the Z–R relationship, termed as M1/NLREG. Both techniques are implemented for a regression of Z over R and for a regression of R over Z. Recall that the latter regression equation should be used for estimation of rain rates from reflectivity measurements. For Methodology 2 (M2), two variants are also defined based on the choice of the model for the shape function \( g(x) \): (i) exponential (M2/EXP) and (ii) gamma (M2/GAM). In addition, it was found useful to test the influence of a threshold on the rain rate with values of \( \text{ThR} \) equal to 0.1, 0.5, 1.0 and 5.0 mm h\(^{-1}\) for both methodologies. Spectra with rain rates less than \( \text{ThR} \) are discarded from the analysis. Finally, the influence of the integration time step is also studied, with values of \( \Delta T \) equal to 1, 2, 5 and 10 min. A full sensitivity test would have had to include the scattering model. However, since our primary interest is for the S-band frequency in the present work, we have simply implemented the Rayleigh scattering model hereafter.

Fig. 2. Example of DSD scaling law adjustment over the 2004 DSD dataset with an integration time step of 5 min and 3 rain-rate thresholds of 0.1, 1.0 and 5.0 mm h\(^{-1}\) from left to right. Top graphs: estimation of the scale parameters \( \alpha, \beta \) through a linear regression of the exponent \( b_n \) as function of the moment order \( n \) (\( b_n \) is the exponent of the power-law model between the \( n \)th order moment of the DSD and the rain rate). Bottom graphs: set of scaled spectra (grey curves) together with the exponential and gamma models that could be fitted for the shape function \( g(x) \).

4.2. Discussion on DSD fittings

Fig. 2 presents examples of DSD fittings for the 5-min integration time step and various ThR values. Table 2 provides a list of part of the DSD model parameters that were adjusted. These elements call for the following comments:

The top graphs in Fig. 2 show that the exponents of the n-th order moments of the DSD as function of the rain rate are well organised with respect to the moment order for n greater than 2. This is an indication of a rather robust estimation of the \( \beta \) parameter. The departure from the straight line observed for lower order moments may be attributed to the measurement noise affecting the small diameters.

Table 2 indicates that the \( \beta \) values are comprised between 0.186 and 0.215, depending on the rain-rate threshold and integration time step considered. This result is consistent with the findings of several authors, including Marshall and Palmer (1948) with \( \beta \approx 0.21 \), and Delrieu et al. (1997) who summarized various DSD exponential model fittings for Mediterranean rainfall. The \( \beta \) parameter increases slightly with the value of \( \text{ThR} \) and remains rather constant as function of the integration time step.

The \( \alpha \) parameter is more variable with values in the range of 0.001 up to 0.140. It tends to decrease when \( \text{ThR} \) increases (this is consistent with the \( \beta \) parameter evolution) and to slightly increase when the integration timestep increases. Such \( \alpha \) values are indicative of the small dependence of the prefactor of the negative exponential model on the rain rate. Again, this fact is consistent with previous findings. For instance, Marshall and Palmer (1948) proposed a constant value for the exponential prefactor: \( N_0 = 8.0 \times 10^3 \text{ mm}^{-1} \text{ m}^{-3} \). Delrieu et al. (1997) report \( \alpha \) values in a very similar range for Mediterranean rainfall.

Regarding the shape function \( g(x) \), Fig. 2 shows that the scaled spectra present a mode for a normalized diameter in the range of 0.2–0.7 mm\((1-\beta)\) \( h^n \). The spread of the scaled spectra is quite large due to the variety of the rain micro-physical processes involved in the season sample considered here. The variability is higher for the highest normalized diameters; it very significantly decreases as the rain-rate threshold is increased. The exponential model clearly fails to provide a correct fitting over the entire normalized diameter range. Its better fitting quality as the rain-rate threshold increases explains the subsequent increase of the \( \kappa_1 \) parameter values (Table 2). The range of \( \kappa_1 \) parameter values \( (1.6 \times 10^3–3.9 \times 10^3 \text{ mm}^{-1} \text{ m}^{-3} ) \) remains however much lower than the value proposed by Marshall and Palmer \( (\kappa_1 = N_0 = 8.0 \times 10^3 \text{ mm}^{-1} \text{ m}^{-3} ) \) and is consistent with exponential DSD model fittings reviewed by Delrieu et al. (1997) for Mediterranean rainfall. Similar comments can be made for the \( \lambda_1 \) parameter values with an observed range of values \( 3.0–3.3 \text{ mm}^{\beta-1} h^\beta \) \( (4.1 \text{ for the Marshall–Palmer DSD model; 3.0–4.0 for the DSD models reviewed by Delrieu et al. (1997))} \). The gamma model fitting is clearly more satisfactory than the exponential model fitting and the values of the \( \mu \) parameter in the range of 2.0–5.0, indicating a marked concave downward shape, are also frequently reported in the literature.

To summarize, the DSD fittings obtained are consistent with previous findings in Mediterranean rainfall. The fittings are very sensitive to the rain-rate threshold and, rather surprisingly, not very much sensitive to the integration time step considered.

<table>
<thead>
<tr>
<th>( \Delta T ) (min)</th>
<th>( \text{ThR} ) (mm h(^{-1}))</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \kappa_1 ) (mm((1+\alpha))m(^{-3}) h(^{-3}))</th>
<th>( \lambda_1 ) (mm((\beta-1)) h(^{-3}))</th>
<th>( \kappa_2 ) (mm((1+\alpha+\mu))m(^{-3}) h(^{-3}))</th>
<th>( \mu )</th>
<th>( \lambda_2 ) (mm((\beta-1)) h(^{-3}))</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.089</td>
<td>0.195</td>
<td>1.59 \times 10^3</td>
<td>2.99</td>
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<td>5.00</td>
<td>7.93</td>
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<td>2.09</td>
<td>5.80</td>
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(\( \alpha, \beta \)) are the scale parameters; (\( \kappa_1, \lambda_1 \)) and (\( \kappa_2, \mu, \lambda_2 \)) are the parameters of the exponential and gamma models of the shape function \( g(x) \), respectively.

Table 3 displays some of the $Z$–$R$ relationships we were able to fit with the various estimation methods and parameters (rain-rate threshold, integration time step) described in Section 4.1. The most striking result in Table 3 is the important diversity of the $Z$–$R$ coefficients fitted over this single dataset: the prefactor takes values in the range of 56 up to 536 and the exponent in the range of 1.43 up to 1.92.

4.3. Discussion on $Z$–$R$ fittings

Table 3 displays some of the $Z$–$R$ relationships we were able to fit with the various estimation methods and parameters (rain-rate threshold, integration time step) described in Section 4.1. The most striking result in Table 3 is the important diversity of the $Z$–$R$ coefficients fitted over this single dataset: the prefactor takes values in the range of 56 up to 536 and the exponent in the range of 1.43 up to 1.92.

As shown in Fig. 3, the M1 correlation coefficients are comprised between 0.87 and 0.90. This indicates a good quality of fit for the models adjusted with however a significant residual variance. Interestingly, this residual

<table>
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<tr>
<th>$\Delta T$ (min)</th>
<th>$\text{ThR} \ (\text{mm h}^{-1})$</th>
<th>M1 LOGREG $Z$ over $R$</th>
<th>M1 LOGREG $R$ over $Z$</th>
<th>M1 NLREG $Z$ over $R$</th>
<th>M1 NLREG $R$ over $Z$</th>
<th>M2 EXP $R$ over $Z$</th>
<th>M2 GAM $R$ over $Z$</th>
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Fig. 3. $Z$–$R$ scattergraphs and power-law models fitted with the logarithmic (continuous lines) and non-linear (dotted lines) regression techniques over the 2004 DSD dataset. Both the $Z$ versus $R$ and $R$ versus $Z$ regression models are displayed for each regression technique. The rain-rate threshold is set to 1 mm h$^{-1}$, i.e. spectra with rain rate less than 1 mm h$^{-1}$ are discarded from the regression analysis. The various graphs correspond to integration time steps increasing from 1 to 10 min.
The variance remains quite steady when the integration time step increases. Although such residual variance behaviour may be partly sampling-size dependant, it seems that time integration from 1 to 10 min does not bring the DSD sample “noise” reduction one should expect.

The M1/LOGREG relationships present prefactor and exponent values which are typical, with however exponents of the \( R \) versus \( Z \) regressions greater than 1.7. The rain-rate threshold influences a lot the regression coefficient values; their evolution as function of the integration time step is moderate. The non-linear regressions lead to coefficients which are much less typical, with prefactors less than 130 and exponents greater than 1.75 for \( \Delta T \leq 5 \) min. Giving more weight to the highest \((Z, R)\) pair values, the non-linear regressions are not sensitive to the rain-rate threshold (a positive point), but they may become very sensitive to the sample size, as illustrated by the discontinuity of the \( Z \)–\( R \) coefficient values between the integration time steps less or equal to 5 min and those obtained for 10 min.

With the M2 methodology, we can observe that the exponents are in a rather narrow range (1.45–1.51). By construction, the exponents are equal for the exponential and gamma shape function models. However, one can notice that the M2/EXP prefactors are more than twice as great as the M2/GAM prefactors! The latter result is consistent with the inadequacy of the exponential model to our DSD spectra mentioned above.

The diversity of the \( Z \)–\( R \) coefficients obtained over this single DSD dataset is quite impressive regarding the influence of the \( Z \)–\( R \) relationship estimation technique. Such a diversity can be attributed partly to the derivation methodology and also to a large part, to the variability of the micro-physical processes involved at the season scale in the dataset.

4.4. Consistency test of the \( Z \)–\( R \) relationships

It is important to assess the impact of such a diversity of \( Z \)–\( R \) relationships in terms of rainfall estimation. For this purpose, we have realized a simple consistency test based on the following procedure:

1) We define reference rain-rate time series \( R_{\text{REF}}(t) \) by a direct estimation of the rain rate from each DSD spectra, according to (3),
2) Similarly, we define reflectivity time series \( Z(t) \) by a direct estimation of the reflectivity from each DSD spectra, according to (2),
3) Then we convert the reflectivity time series into rain-rate time series \( R_{ZR}^*(t) \) using the \( Z \)–\( R \) relationships obtained with the various techniques discussed in Section 4.3. Note that, concerning methodology M1, we use the regression equations of \( R \) over \( Z \).

4) Finally, we are able to compute various assessment criteria between the \( R_{ZR}^*(t) \) and \( R_{\text{REF}}(t) \) rain-rate time series. We report hereafter the results obtained in terms of the relative error on the total rain amount and of the Nash coefficient between the reference and estimated rain-rate time series. Recall that the Nash criterion, a widely used assessment criterion in hydrology, is defined as:

\[
N = 1 - \left[ \frac{\sum (R_{\text{REF}}(t) - R_{ZR}^*(t))^2}{\sum (R_{\text{REF}}(t) - \bar{R}_{\text{REF}})^2} \right]^{1/2}
\]

where \( \bar{R}_{\text{REF}} \) is the average value of the reference rain-rate time series. Recall that \( N = 1 \) denotes a perfect agreement between the two series; \( N \) is equal to zero when the estimator provides estimates as poor as the simple average of the reference values.

Fig. 4. Results of the sensitivity test concerning the influence of the \( Z \)–\( R \) relationship estimation methodology on the estimation of the rain parameters: total rain amount (top graph) and Nash coefficient of the rain rate time series (bottom graph). In top graph, the horizontal line at about 280 mm represents the reference total rain amount. See text, Section 4.4, for details.
The results displayed in Fig. 4 can be summarized as follows. Methodology M1 is clearly superior to M2 for both criteria. In other words, with respect to the consistency test performed and the DSD dataset considered here, it seems much better to estimate the $Z-R$ relationship with a non-linear regression technique on the ($Z$, $R$) scattergraph, rather than using the DSD modelling approach. In the details, one can notice on Fig. 4 that:

- Concerning M1, the total rain amount is estimated to within ±6% by the regression techniques whatever the integration time step. Using the highest rain-rate threshold (5 mm h$^{-1}$) has however a detrimental effect and such high rain-rate thresholds should therefore be avoided. When looking at the Nash criterion, a slight superiority of the NLREG regressions over the LOGREG regressions can be evidenced, even if the NLREG $Z-R$ relationship parameters (Table 3) were found to be less “typical”.

- Concerning M2, the total rain amount is fairly underestimated by M2/EXP (−46% up to −25%, depending on the integration time step) and significantly overestimated by M2/GAM (14 up to 20%, depending on the integration time step). The failure of the M2/EXP approach is event more impressive with the Nash criterion, while the M2/GAM approach is about as good as the M1 approach for this criterion. These results indicate the sensitivity of M2 to the shape function fitting: the exponential model is clearly not flexible enough with its two parameters, but the gamma model also partly fails. A proposition could be to use non-parametric $g(x)$ functions, estimated by averaging the scaled DSD spectra. Furthermore, in contradiction with the results obtained with M1, it can be noted that the M2 performance is slightly better when considering the highest rain-rate threshold of 5 mm h$^{-1}$. This result may also be related to the fitting quality of the shape function which increases when the rain-rate threshold increases (e.g. Fig. 2).

4.5. Insight on the inter-event DSD variability

The $Z-R$ adjustment procedure and the consistency test were repeated at the event time scale in order to assess the inter-event DSD variability and its impact on rain estimation. To illustrate the results obtained, we display in Table 4 the Nash criterion values for the various $Z-R$ estimation procedures when considering event-fitted and seasonal $Z-R$ relationships. While one would expect a significant improvement with the use of event-fitted $Z-R$ relationships, the results show that improvements are far from being systematically observed. In several instances, there is on the contrary a detrimental effect in adapting the $Z-R$ relationship to each rain event. There is also no clear trend regarding a fitting procedure which would do systematically better or worse than the other ones. In our opinion, these rather paradoxical results may be explained by two factors: (1) the intra-event DSD variability is dominant with respect to the inter-event variability; (2) in some instances, the event-fitted $Z-R$ relationships may lack of robustness and then be less efficient than the seasonal $Z-R$ relationships. This finding motivates an in-depth analysis of the intra-event DSD variability, the subject of Section 5.

5. Intra-event DSD variability and link with the storm structure

As mentioned in Section 2, the 27–28 October 2004 rain event is the most intense case among the available dataset with a rain total of about 100 mm in about 16 h (Fig. 5). The event was organised as a series of rain bands which major axis was oriented in the SE–NW direction, with a global moving direction toward NE. First, we analyse the DSD at ground level and, in a second part, we try to establish a link between the DSD at ground level and the storm structure, as sensed with the Bollène volume-scanning weather radar system.
5.1. Analysis of the DSD dataset observed at ground level

The rain event was mostly active and intense between 1800 UTC and 0000 UTC (Fig. 5) with rain bursts exceeding frequently one-min rain rates of 20 mm h$^{-1}$ and reaching a maximum value of 78 mm h$^{-1}$. The total rain amount is equal to 67 mm during this period. In the second part of the night, from 0000 UTC to 0600 UTC, rainfall was more continuous with moderate rain rates intensities (5–20 mm h$^{-1}$) and a total rain amount of 23 mm. Fig. 5 displays the time evolution of several DSD moments (total number of raindrops per cubic meter, mean and standard deviation of the diameter, rain rate). This information was used to perform an empirical separation of several phases in the rain event, numbered from 1 to 7 in Fig. 5.

Phases 1 and 7 correspond to the beginning and the end of the rain event: they are characterized by isolated rain peaks of short duration. In both phases, the $N_t$ parameter is low (less than 500 m$^{-3}$) and the mean and standard deviation of the diameters are very noisy. The most intense rainy period between 1800 and 0000 UTC was divided into 2 phases. Phase 2 lasted about 3 h. It is
characterized by two rain peaks with rather high $N_t$ values (between 1000 and 2000 m$^{-3}$), mean diameters between 0.6 and 1.0 mm and diameter standard deviations varying rapidly in a range of 0.2–0.6 mm. Although rain rates and total rain amounts are almost as high as in phase 2, phase 3 (3 h) has a very different DSD moment structure since the number of drops is less than 1000 m$^{-3}$, the mean diameter increases and remains roughly equal to 1 mm; the diameter standard deviation also increases and varies in the range of 0.4–0.6 mm. During the second part of the night, three different phases were identified. Phase 4 (100 min) is characterized by low rain rates (5 mm h$^{-1}$) during 2 h with low $N_t$ values (about 250 m$^{-3}$ in average), a high mean diameter (about 0.8 mm in average) and a diameter standard deviation of about 0.35 mm. A new regime change occurs in phase 5 (100 min) with a trapezoidal rain peak presenting a sill of about 15 mm h$^{-1}$ that lasted about 40 min. This was associated with a very significant increase of the $N_t$ parameter (about 2000 m$^{-3}$), a mean diameter between 0.6 and 0.9 mm, following the rain-rate evolution, and a low diameter standard deviation of about 0.2 mm. Phase

![Fig. 6. Results of the DSD scaling analysis for the 5 main phases of the 27–28 October 2004 rain event. The normalized 5-min DSD spectra are displayed in grey together with the non-parametric (simple average, dash-dotted line) and the Gamma (dotted line) fittings for the shape function $g(x)$. The number of spectra, the scale parameters ($\alpha, \beta$), the coefficients ($\kappa_2, \mu, \lambda_2$) of the $g(x)$ Gamma fitting and the corresponding $Z$–$R$ relationship are indicated in each single frame.](image-url)
leviatal" at about 3 km in phase 2 and 3 and it reflects as function of the DSD moments. The 35-dBZ results concerns the maximum altitude of the 35 dBZ radar for an exhaustive description. One of the most significant disappointing. The readers are referred to Chapon (2006) attempts to quantify the correlation between radar-derived between the radar and DSD-sensor signals, the various elements are reported in this figure as well.

5.2. Link with the storm structure

Fig. 7 displays the Bollène radar reflectivity time series as function of the altitude above ground at the DSD-sensor site. The rain-rate time series and the DSD phasing derived in Section 5.1 from the DSD measurements are reported in this figure as well.

Although there is a clear qualitative consistency between the radar and DSD-sensor signals, the various attempts to quantify the correlation between radar-derived and ground-based DSD-derived parameters were rather disappointing. The readers are referred to Chapon (2006) for an exhaustive description. One of the most significant results concerns the maximum altitude of the 35 dBZ radar reflectivity as function of the DSD moments. The 35-dBZ level is at an altitude of about 3 km in phases 2 and 3 and it decreases quite regularly in the second part of the rain event. A positive and relatively high correlation is observed with the DSD-derived mean diameter (correlation coefficient of 0.75) while there is no correlation with the \( N_i \) parameter (−0.16) and moderate correlations with the DSD-sensor rain rate and reflectivity parameters (0.43 and 0.47, respectively). These results confirm the control of the vertical extension of rainfall on the size of raindrops at ground level. The correlation analyses also led us to the conclusion that the radar-DSD-sensor distance (57 km) is too high to allow an in-depth quantification of the link between the rain structure in the atmosphere and at ground level, owing to the very distinct sampling properties of the two types of sensors at such a range. This situation will be improved in the future with the implementation of DSD sensors at closer ranges from the Bollène and Nîmes radars as part of the new observation strategies of OHM-CV.

However, from an operational perspective, improving radar QPE for such (and greater) radar ranges is particularly crucial. Therefore, as a complementary insight to the detailed correlation analyses mentioned above, we have applied a rain-typing procedure which separates automatically the convective and stratiform regions from 3D weather radar data. The details of the algorithm are explained by Delrieu et al. (in preparation). Basically the Steiner et al. (1995) algorithm, for detection of convective rainfall, and the Sanchez-Diezma et al. (2000) algorithm, for detection of the bright band (as a signature of stratiform precipitation), were adapted and endowed with a decision tree. The time steps for which a convective detection is obtained are marked in Fig. 7. Due to the operating protocol and the 27–28 October 2004 rain event
characteristics (low vertical extension), the stratiform algorithm was not able to detect any consistent bright bands for this event in the vicinity of the DSD sensor. It is noteworthy that the two rain peaks of phase 2 and the entire phase 3 are stamped as convective together with a number of isolated time steps in phases 1, 6 and 7. The automatic classification obtained here on the basis of reflectivity measurements (i.e. the 6th order moment of the automatic classification obtained here on the basis of number of isolated time steps in phases 1, 6 and 7. The automatic classification obtained here on the basis of reflectivity measurements (i.e. the 6th order moment of the DSD is satisfactory and is consistent with the time evolution of rain rates displayed on the Fig. 5. We find however that the variability of the DSD-derived $Z$–$R$ relationships, between parts of the event which were all classified ‘convective’ (phases 2–3) or on the contrary all classified ‘non-convective’ (phases 4–5) is high. This reveals the complexity of the micro-physical processes within a given type of rain and we may ask whether two or three average $Z$–$R$ relationships will be more powerful than a single one to account for such a variability. Clearly, the available dataset does not allow a robust inference for such typed $Z$–$R$ relationships and long-term observations need to be performed to assess this point.

6. Summary and conclusion

This paper presents the analysis of a DSD dataset collected in Mediterranean rainfall within the Cévennes-Vivarais Mediterranean Hydro-meteorological Observatory. Two experimental design characteristics have actually limited the impact of this preliminary investigation conducted in 2004: (i) the Bollène radar was operated in volume-scanning mode for only one rain event during the measurement period, (ii) the DSD sensor was located too far from the radar site (57 km) for an in-depth characterization of the links between the rain parameters in the atmosphere and at ground level. Therefore, the analysis was mostly dedicated to the characterization of the DSD variability at ground level and its impact on the reflectivity — rain-rate conversion ($Z$–$R$ relationship). Two concurrent approaches for estimating the $Z$–$R$ relationship from DSD measurements were presented. The first one is based on application of non-linear regression techniques on the scattergraphs of the $(Z, R)$ pairs derived for each DSD spectrum. The second one uses as an intermediate step a fitting of a DSD model. We considered here the scaling law formulation for the DSD initially proposed by Sempere Torres et al. (1994). The two methodologies, with a number of variants, were implemented over the dataset obtained by grouping the DSD spectra of five rain events, corresponding to 75 h of rain. The DSD fittings obtained were found to be consistent with previous findings for Mediterranean rainfall. However, the $Z$–$R$ relationships derived for this single DSD dataset were found to be very diverse, strongly dependent on the fitting methodology and the threshold considered on the rain rate. This result is certainly due to the heterogeneity of the rain processes at the season scale. On the other hand, a relative stability of both the DSD models and the $Z$–$R$ relationships was observed as a function of the integration time step (1 to 10 min). This indicates a significant temporal organisation of these rain processes. A consistency test of the $Z$–$R$ relationships was then proposed to assess the most reliable estimation procedures. For this purpose, the DSD-derived rain-rate time series were used as reference and the various $Z$–$R$ relationships allowed an estimation of rain-rate time series from the DSD-derived reflectivity time series. It was shown that the regression techniques were very significantly better than the methods using the DSD modelling step to estimate both the total rain amount and the rain-rate time series. This result is rather natural since the regression techniques work directly with the DSD moments of interest. It needs to be confirmed with real radar data compared with ground truth. It does, however, serve to warn us that $Z$–$R$ relationships derived from DSD models may be far from optimal to estimate rainfall parameters. The same consistency test was used to show that event-fitted $Z$–$R$ relationships do not systematically improve the test scores compared to the seasonal $Z$–$R$ relationship. This result supports the idea that the intra-event DSD variability is dominant. This fact has motivated an in-depth analysis of the 27–28 October 2004 rain event. The DSD scaling formalism allowed us to show that the rain event is organised into a series of different phases, each one presenting very stable DSD scale and shape parameters over several successive hours, with abrupt transitions from one phase to the next. A rain-typing algorithm applied to the 3D reflectivity data observed concomitantly with the Bollène radar was able to recognise the two most intense phases of the rain event as “convective”.

Work on this subject will continue within OHM-CV along the following lines. It seems important to enhance and systematize the documentation of DSD variability in the Cévennes-Vivarais region. Therefore several DSD sensors, coupled with rain gauges, will be installed in a number of locations representative of various parts of the region (mountain, transition, plain) and located at various ranges from the radar sites. Such sites will be used in conjunction with the 3D-scanning radar datasets to establish $Z$–$R$ climatological relationships with or without radar rain typing. Apart from this regional radar QPE objective, detailed DSD instrumentation and analyses will be performed at close range (20 km or less) for both radars to improve our understanding on the
dynamic link between the DSD in the atmosphere and at ground level. The impact of DSD space–time variability on runoff and erosive processes will be also studied in agricultural land plots of an experimental site called “Le Pradel”.

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References


